# INSTABILITY OF INTERNAL WAVES FROM A CYLINDER IN A SHEAR FLOW WITH A LARGE RICHARDSON NUMBER

V. I. Bukreev

UDC 532.593

A dynamic system is considered that consists of two fluids with densities $\rho_1$ and $\rho_2 < \rho_1$ moving in plane-parallel motion above a horizontal bottom at the constant velocities $u_1$ and $u_2$ at the depths $H_1$ and $H_2$ (the subscript 1 refers to the lower and 2 to the upper layer). There is an interlayer with the characteristic thickness $\delta \ll H_1, H_2$ in the zone of fluid contact in which the density and velocity change smoothly from one constant value to the other. According to the criterion [1]

$$\mathrm{Ri} = \varepsilon g\delta/(u_2 - u_1)^2 > \mathrm{Ri}_*, \quad \varepsilon = \rho_1/\rho_2 - 1$$

(g is the acceleration of gravity) the system is considered stable. The question of whether waves generated translationally by a moving cylinder can become unstable therein is clarified experimentally. For the sequel it is possible to take $\mathrm{Ri}_* = 0.25$ since refinements of this quantity [1] for Ri values utilized in tests are insubstantial.

At this time there is a large number of theoretical papers (sufficiently complete information thereon can be found in [1-5]) predicting the possibility of a loss of stabilty for other reasons than which has been reflected in the mentioned criterion. Even small perturbations described by linear theory can, say, lose stability because of the existence of negative energy in the wave system [5]. The possibility of the loss of stability under the action of finite amplitude perturbations increases significantly [2]. Out of the experimental stability investigations for stratified liquid shear flows executed earlier, [2, 4, 6] can be noted as being closest to the present paper.

Tests were performed in an installation whose diagram is presented in Fig. 1. It was a 5 m long, 0.2 m wide, and 0.6 m high channel with a horizontal bottom and organic glass walls. First a solution of glycerine in water with the density $\rho_1 = 1.013$ g/cm³ and viscosity $\nu_1 = 0.0118$ cm²/sec was poured into the channel. Then distilled water with $\rho_2 = 1$ g/cm³ and $\nu_2 = 0.0105$ cm²/sec was slowly poured through porolon filters floating on the surface. The upper layer was later set into motion by a propeller pump at the velocity $u_2$. The velocity of the lower layer could be considered zero with good accuracy (See Fig. 2).

A perforated tube, a gravel filter 1 and a horizontal plate 2 arranged slightly above the interface boundary were used to equilibrate the stream at the entrance to the channel working section. A plate 7 bent backward from below, two meshes 5, and a cylindrical fairing 6 were installed at the channel exit. All this assured homogeneity of the flow in the longitudinal direction with variations in $u_2$ of not more than 5% per 1 m of length and a level of uncontrollable perturbations for which visually distinguishable waves appeared at the interfacial boundary only for Ri < 0.8 (compare [6]).

The velocity profile u(y) in the coordinate system displayed in Fig. 1 is determined from trajectories of particles of about 1 mm in size fabricated from a mixture of rosin and paraffin. The particle trajectories were recorded by a movie camera and the velocity was calculated by the increment in the coordinate $\Delta x$ in a known time $\Delta t$. A correction was introduced for optical distortions during filming. The velocity profile used later is presented in Fig. 2. Each experimental point is obtained by taking the average over ten particles, and the horizontal segments indicate the intervals in which the results of measurements taken individually were located. The ordinate of the free surface is marked by a triangle. The density was determined by suspension of specimens taken from the points under consideration by using a medicine dropper with 0.2 mm hole diameter. The density profile used later is presented in Fig. 2 (line 2).

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 85-89, September-October, 1988. Original article submitted June 9, 1987.

  

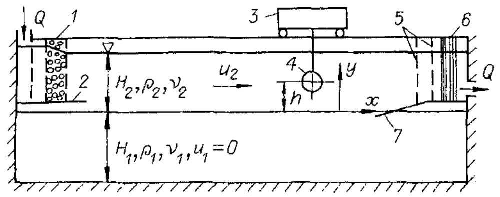


Fig. 1

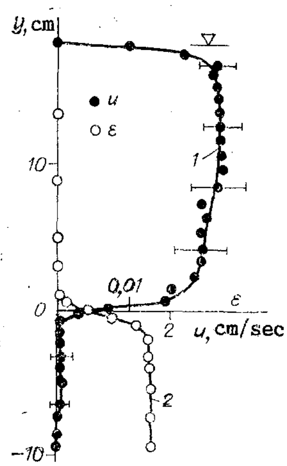


Fig. 2

By using trolley 3 the cylinder 4 of D = 2 cm diameter was moved perpendicularly to its axis along the trajectory y = h = const at the velocity

$$U = \begin{cases} U_0[1 - \exp(-t/\tau_1)] & \text{for} \quad 0 < t < T, \\ U_0 \exp(-t/\tau_2) & \text{for} \quad t \geqslant T, \end{cases}$$

where t is the time and $U_0$, $\tau_1$, $\tau_2$ and T are constants. Here $\tau_1$, $\tau_2$ were of the order of magnitude of 0.2 sec while T varied between the limits 15-150 sec so that the cylinder traversed its trajectory almost entirely within the uniform motion mode.

Studied in the tests was the behavior of that equal-density surface $\rho_0$ = const, $\rho_2 < \rho_0 < \rho_1$ that would appear in the photographs as a sharp boudnary between light and dark images during filming with the lower layer colored by ink. In the unperturbed state the location of this surface corresponds to the value y = 0 in Fig. 2. Both the main flow and the waves generated by the cylinder were considered stable if the mentioned boundary remained smooth during visual observation and on the photographs. Either at least partial destruction of the waves or fluid mixing between the layers was the criterion for loss of stability.

The system parameters $\varepsilon = 0.013$, $\delta/H_1 = 0.08$ and $\delta/H_2 = 0.1$ did not vary in the tests while the parameter Ri took on two values Ri = 3.1 exceeding $Ri_*$ by 12.4 times, and Ri → ∞ (the case of layers at rest in the unperturbed state). The δ entering in these parameters is defined as the distance between the points at which the tangent to the profile ρ(y) intersects the lines $\rho = \rho_1$ and $\rho = \rho_2$ at $\rho = \rho_0$. The most substantial perturbation parameters varied within the ranges $2 \leq |h/D| \leq 6$, $0 \leq Fr = (2 + \varepsilon)U_0^2/\varepsilon gR \leq 81$, $400 \leq Re = U_\infty D/\nu_2 \leq 4000$. Therein R = D/2, $U_\infty = |U_0|$ during cylinder motion in the lower layer and $U_\infty = |U_0 \pm u_2|$ in the upper. The sign of the parameter $u_2/U_0$ is also of importance in the shear flow; under definite conditions stable waves become unstable for just one change in the sign of this parameter.

The moving cylinder induces perturbations of two kinds. On the one hand, the fluid particles while flowing over it acquire vertical accelerations resulting in the occurrence of regular internal waves in the stratified medium. On the other, a hydrodynamic wake is formed behind the cylinder which was entirely turbulent for the Re values considered in the tests although quite definite ordered (coherent) structures also existed therein. The wake can cause fluid mixing between the layers upon reaching the interfacial boundary. In these tests such a pattern was observed for Ri → ∞, h/D = 4, Fr = 81, say at distances from the cylinder exceeding two hundred of its diameters. Indirectly the wake also induces a wave-type contribution to the perturbation, magnifying it [7].

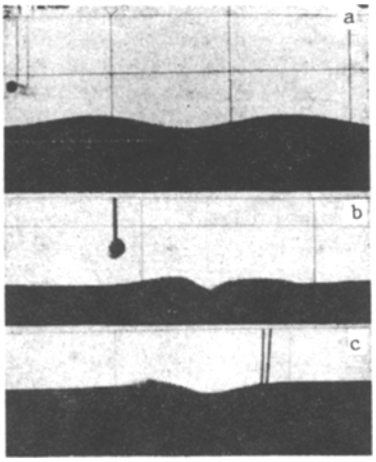

Fig. 3

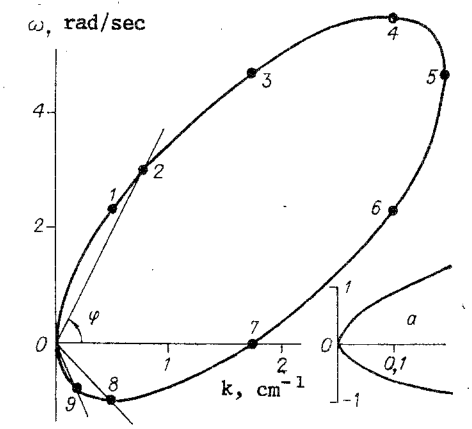


Fig. 4

A separate study of the system reaction to the perturbations of the two kinds mentioned turned out to be possible because domains of values of Ri, h/D and Fr exist where either one or the other predominates. Thus, for the considered Ri and h/D the direct influence of the wake was felt only for Fr > 50, while the sign-varying waves excited by the cylinder itself became negligibly small in the zone of wake emergence on the interfacial boundary for Fr > 20. The results of those tests in which the wave-type perturbations predominated will be discussed in greater detail later.

Waves from the cylinder are stable in the fluid at rest ($Ri \to \infty$) in the whole range of parameters. The highest value of the parameter $\beta = 2\pi a/\lambda$, characterizing the wave steepness according to [2], reached 0.3 (a is the amplitude and $\lambda$ the wavelength). Computations on the basis of a nonlinear mathematical model [2] confirm the possibility of the existence of stable waves of such significant steepness in a fluid at rest.

In a shear flow with Ri = 3.1 wave-type perturbations are stable for all Fr if $|h/D| > 5$. In the $2 \leq |h/D| \leq 4$ range both stable and unstable waves are observed. An example of the stable waves being formed for h/D = 3, Fr = 3.37, $U_0 = -4.62$ cm/sec is presented in Fig. 3a ($\beta = 0.19$), where the free surface is denoted by the triangle, and the spacing between divisions of the coordinate grid is 20 cm along the horizontal and 10 cm along the vertical.

Unstable wave-type perturbations being observed for Ri = 3.1, h/D = 2, Fr = 0.7, $U_0 = -2.11$ cm/sec are shown in Fig. 3b. Intense vortices that absorb almost all the perturbation energy are formed in the wave troughs in this mode. As h/D increases the relative fraction of perturbation energy going over into the regular waves increases continuously so that the waves become stable for h/D > 5.

Still another example of wave-type perturbation instability being observed during cylinder motion in the lower layer in the direction $u_2$ (Ri = 3.1, h/D = −3, Fr = 2.43, $U_0$ = 3.92 cm/sec) is presented in Fig. 3c. In this mode the wave destruction occurred on its trailing front.

The question of whether the wave-type perturbation instability is associated with some singularities of the operator of the system under consideration, particularly, its linear operator, is legitimate. An important characteristic of the linear operator is the dispersion relation $\omega(k)$ ($\omega$ is the circular frequency and k is the wave number). For a shear flow that differs somewhat from the velocity profile under consideration, an analysis of the dispersion relation is executed in [5]. The dispersion relation for the first slow mode in the system being studied is presented in Fig. 4. It is valid for inviscid fluids. The correction for the influence of viscosity can be introduced by means of formulas from [8]. Analysis showed that this correction is substantial for large k while it did not exceed 5% for the modes realized. The fragment a in Fig. 4 contains the domain of small values of k in a larger scale. The influence of the fact that the layers in the tests were of finite depth is felt in this domain.

There is a number of characteristic points on the dispersion curve presented. The phase velocity $c = \omega/k$ at the point 0 (the origin) equals the group velocity $c_* = d\omega/dk$

and the limit velocity of infinitesimal sinusoidal wave propagation in a system with finite layer depth, while in the presence of velocity shear it takes on two values $c_m^+$ and $c_m^-$ (in tests $c_m^+ = 13.9$ cm/sec and $c_m^- = -10.7$ cm/sec). Only conoidal, solitary, intermittent waves and smooth bora which linear theory does not describe can be propagated at a velocity exceeding the limit in the system under consideration.

At the point 5 the group velocity becomes infinite and unstable according to the Kelvin–Helmholtz mechanism. The phase velocity lies between $u_1$ and $u_2$ on the arc of the dispersion curve 3-5-7. Critical layers through which small perturbations do not pass in practice exist in the diffused zone for perturbations with phase velocities from this range.

For the sequel, the point pairs 4, 8 and 1, 6 in which the group velocity equals $u_1$ (zero in the selected coordinate system) and $u_2$, respectively, are of special interest. It is interesting to note the definite symmetry in the arrangement of the eight points marked. If points corresponding to the mentioned singularities in the group velocity are excluded, then this symmetry is spoiled.

To make a judgement about precisely what stationary waves are excited in the system by a given perturbation, the characteristics of this perturbation must be constructed in the domain $(\omega, k)$. The equation of the characteristic is $\omega_* = U_0 k$ for translational cylinder motion. The point of its intersection with the dispersion curve in Fig. 4 governs the length, frequency, phase and group velocities of the first slow-mode waves excited by a cylinder. Three lines $\omega_*(k)$ corresponding to illustrations in Fig. 3 are drawn in Fig. 4.

The line 08 passes exactly through the singular point 8 of the dispersion curve and an intensive process of loss of stability illustrated in Fig. 3b is observed. The line 02 intersects the dispersion curve on the section between the singular points 1 and 4 near the point 1. In this case the instability illustrated in Fig. 3c also holds. The line 09 intersects the dispersion curve outside the intervals in which c or $c_*$ lie between $u_1$ and $u_2$ and the waves generated by the cylinder are stable even for large $\beta$ (Fig. 3a).

Therefore, waves from a cylinder in a shear flow can be destroyed even in the case when the main flow is stable according to linear theory. The predisposition to destruction is governed here not only by the wave steepness but also by what domain of parameters determined by the dispersion relation their excitation occurs in. In addition to the known information about the singularities of wave behavior in the domain where their phase velocity lies between the upper and lower layer velocities, attention should be turned to the elevated instability of the wave perturbations induced by the cylinder under conditions when their group velocity lies between the upper and lower layer velocities.

The author is grateful to A. V. Gusev and E. F. Vedernikov for the main contribution to the realization of the test program.

LITERATURE CITED


1. J. S. Turner, Buoyancy Effects in Fluids, Cambridge Univ. Press, NJ (1973).
2. S. A. Thorpe, "On the shape and breaking of finite amplitude internal gravity waves in a shear flow," J. Fluid Mech., 85, Pt. 1 (1978).
3. R. Ahmed and S. Banerjee, "Finite amplitude waves in stratified two-phase flow: transition to slug flow," AlChE J., 31, No. 9 (1985).
4. C. C. Koop and B. McGee, "Measurements of internal gravity waves in a continuously stratified flow," J. Fluid Mech., 172, 453 (1986).
5. M. I. Derkach and A. G. Stetsenko, "Shear flow stability in the upper layer of the ocean in the presence of negative energy waves," Morsk. Gidrofiz. Zh., No. 1 (1987).
6. T. Andow, K. Hanawa, and Y. Toba, "Experimental study of internal waves in a stratified shear flow," J. Oceanogr. Soc. Jpn., 37, 179 (1981).
7. V. I. Bukreev, "Waves on the interfacial boundary of two fluids of different densities, generated by the motion of a circular cylinder and a symmetric wing," Prikl. Mekh. Tekh. Fiz., No. 1 (1980).
8. V. I. Bukreev, A. V. Gusev, and I. V. Sturova, "Internal wave generation under joint translational and vibrational motion of a cylinder in a two-layered fluid," Prikl. Mekh. Tekh. Fiz., No. 3 (1986).